For high school and beyond

A textbook well suited for study as well as for self study.
All subjects for high School are included – and more.
This textbook is meant to be:

- *Your math companion for upper secondary school, high school, and the first semester(s) of study.*
- *Textbook for high school or similar as well as the first semester(s) of study.*
- *Textbook which should be supplemented by the accompanying exercise book "WorldMathBook, Exercises" as well as a formula collection.*

The book is independent of which formula collection is used. You can use the book without a formula collection, but it will be harder.
The book is also independent of using a calculator or a calculator program. You can use the book without a calculator, but it will be much harder.
The demands for a calculator/calculator program are:
Part 1. All calculators/programs.
Part 2. Calculators/programs with functions, - almost all kinds have that. Part 3. Advanced calculators/programs able to differentiate and integrate and for plotting curves in diagrams.
Part 4. The setup of vectors is not beneficial using calculators/programs, but if so, an advanced calculator/program is necessary.
Part 5. Advanced calculators/programs for regression.

We start with the four basic arithmetic operations, and finish in the first or second semester of the study for bachelor or candidate.
The language is clear, understanding is in focus, and technical terms are explained.

Overall Content:
This textbook (as well as the exercise book) is divided into five parts:

1. **Basics**
2. **The coordinate system in the plane (2D) and functions**
3. **Differentiation and integration**
4. **Vectors**
5. **Statistics** (including Probability)

Also, at the end we present "Numbers and **Complex numbers**", and some "Rarely used proofs and calculations".
Finally, we present a detailed **Subject index**.

Content in details:

Part 1. Basics

Number system
The four basic arithmetic operations: Sum, Difference, Product, Division
Fractions (Quotients)
Percent and Percentage point
Calculation with letters (algebra)
Parenthesis, Square rules, Square root
Exponentiation
Equations, Second degree equations, Higher degree equations, Two equations with two unknowns
Functions and proportionality
Intervals and inequalities
Imaginary numbers, briefly

Part 2. The coordinate system in the plane(2D) and functions

The coordinate system and distance, The straight line, The parabola, Polynomials
Functions and the four basic arithmetic operations, Composite functions, Inverse functions
The right triangles
The circle
Sine, Cosine and Tangent
Radian, Angle, Arc length, Survey
The sine function and the sine oscillation
The not right-angled triangles (arbitrary triangles)
Proof of the sine-relation and the cosine-relations
Exponential functions
Logarithm functions: log 10-logarithm, natural logarithm: ln (log e)
Other functions
Hyperbola, Third degree polynomial function, Fourth degree polynomial function, Fractional polynomial function, A special third degree polynomial function, Partly defined functions

Part 3. Differentiation and Integration

Introduction
Differential calculus, Proofs of differential calculus 1
The horizontal line, The straight line, The parabola, The square rootfunction, Polynomials, The natural exponential function, The naturallogarithm function
Notations

Differentiation and the four basic arithmetic operations
Sum, Difference, Product, Division
Differentiation of composite functions
Proofs of differential calculus 2
The e^{kx} function, The exponential function, The sine function, The cosine function, The tangent function
Survey
Differentiable, non-differentiable
Integral calculus
Survey and Notations
Integration and the four basic arithmetic operations
Sum, Difference, Product
Integration by substitution
Integration by parts
The specific integral
Areas, Volumes, Guldin's rules, Curve length
Differential equations
Typical differential equations, The logistic differential equation
Slope fields
Functions of two variables
Ways of expression, 3D figures
The gradient

Part 4. Vectors
2D vectors in the plane
Basics, Special vectors, Computations, Angle, Projection, Determinant, Area and angle, The parametric equation for a straight line, Distance point-line
Polar coordinates in 2D
Vector functions (parametric curves) in 2D
The vector function for a straight line, The vector function for a circle.
Differentiation of vector functions: the line, the circle, Double points
3D vectors in the space
Distance point-point, Cross product, Angle between vectors, Area, Equation of a plane, Distance point-plane, The straight line in the space, Distance between skewed lines, Distance point-line, Distance between two parallel planes, Angle between two planes, Angle between line and plane
The sphere

Part 5. Statistics

Data (Observations), Non-grouped data, Grouped data
The normal distribution, variance and standard deviation
Goodness of fit (Chi to the power of two - testing)
Regression, Linear - , Power - , Exponential -
Probability and combination, Introduction, Theory, Examples
Binomial distribution, random sample, and confidence interval
Notations and technical terms

Brief on set theory
Natural numbers, whole, rational, irrational, real, imaginary numbers
Complex numbers, rectangular, polar, exponential
Rarely used proofs and calculations:

- Proof of Pythagoras theorem
- Proof of factorization of a second degree polynomium
- Division of polynomials
- Showing the formulas for permutation and combination
- Proof of product and division of complex numbers in the polar and
- the exponential form

Trial pages 24, 68, 217

Percent

Percent means ”out of a hundred”, which means a fraction with 100 as the denominator.

$\frac{1}{2}$ means 1 out of 2. If we multiply by 50 in the numerator and denominator we get

$\frac{50}{100}$ or 50 out of 100 or 50%. In brief:

$\frac{50}{100} = 50\%$

Examples

$\frac{1}{5} = \frac{20 \cdot 1}{20 \cdot 5} = \frac{20}{100} = 20\%$

$\frac{1}{8} = \frac{12{,}5 \cdot 1}{12{,}5 \cdot 8} = \frac{12{,}5}{100} = 12{,}5\%$

$\frac{1}{4} = \frac{25}{100} = 25\%$

and as a decimal number

$\frac{1}{2} = \frac{50 \cdot 1}{50 \cdot 2} = \frac{50}{100} = 50\% = 0{,}5$

$\frac{1}{4} = \frac{25 \cdot 1}{25 \cdot 4} = \frac{25}{100} = 25\% = 0{,}25$

$\frac{3}{4} = \frac{25 \cdot 3}{25 \cdot 4} = \frac{75}{100} = 75\% = 0{,}75$

$\frac{3}{8} = \frac{12{,}5 \cdot 3}{12{,}5 \cdot 8} = \frac{37{,}5}{100} = 37{,}5\% = 0{,}375$

Percent is out of a hundred. A decimal number is out of one.

1 is one whole. 100% is also one whole.

$1 = \frac{100}{100} = 100\%$

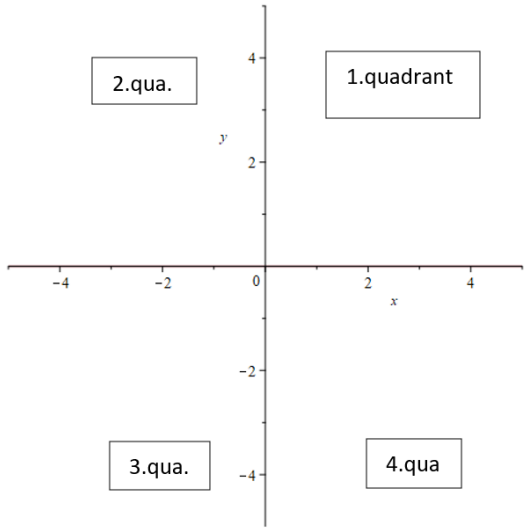
Part 2. The coordinate system in the plane (2D) and functions

The coordinate system and distance

We live in a world of three dimensions, we call it the space and it consists of length, width, and height.

If we work in two dimensions, we call it the plane, and it consists of two directions for instance horizontal and vertical. We may also call the directions for axis. Then we have the *first-axis* and the *second-axis*; or in more technical terms: The *abscissa* and the *ordinate*, both from Latin. Abscissa means ”out (ab) from here (cis)”, which may be pictured by standing at the starting point and looking horizontally at the horizon. The ordinate means the ordinary, which is vertical (all other directions would not be ordinary).

In mathematics we often use the words x-axis and y-axis,

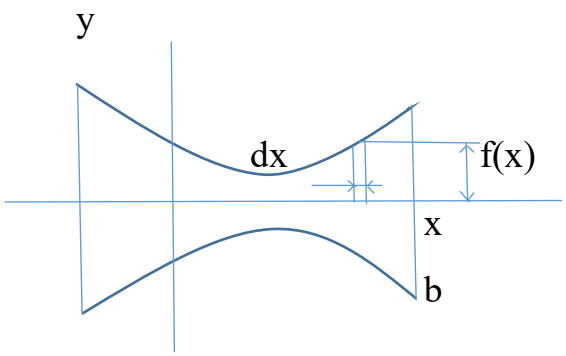


but they can be called other things. In physics the first axis could be t for time, and the second axis could be v for velocity (velox in…

Volumes

We can rotate a 2D area around the x or y-axis and have a 3D volume.

The formula for rotation around the x-axis derives



If we rotate our infinitesimally thin strip around the x-axis we have a micro cylinder. A macro cylinder has the volume

$V = \pi \cdot r^2 \cdot l$ l for length

for our micro cylinder the volume is

$dV = \pi \cdot f(x)^2 \cdot dx$

by integration (gathering all micro cylinders) from a to b

$V = \pi \cdot \int_a^b f(x)^2 \, dx$ the rotation volume around the x-axis

Thus, the volume can be calculated when we have an expression of the function, which informs how the radius varies.
